復旦大學
FUDAN UNIVERSITY

Applied Math Ph.D. Seminar

On the long-time asymptotics of the modified Camassa-Holm equation in space-time solitonic regions

Speaker: Gaozhan Li (Fudan University)

Time: 2022-10-27, 16:10 to 17:00

Location: Rm 1801, Guanghua East Tower

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Abstract: We study the long time asymptotic behavior for the Cauchy problem of the modified Camassa-Holm (mCH) equation in the solitonic regions. Our main technical tool is the representation of the Cauchy problem with an associated matrix Riemann-Hilbert (RH) problem and the consequent asymptotic analysis of this RH problem. Based on the spectral analysis of the Lax pair associated with the mCH equation and scattering matrix, the solution of the Cauchy problem is characterized via the solution of a RH problem in the new scale (y,t). Further using the $\overline{\partial}$ generalization of Deift-Zhou steepest descent method, we derive different long time asymptotic expansion of the solution $u(y,t)$ in different space-time solitonic regions of $\xi = y/t$. We divide the half-plane $\{(y,t) : -\infty < y < \infty,\ t > 0\}$ into four asymptotic regions: The phase function $\theta(z)$ has no stationary phase point on the jump contour in the space-time solitonic regions $\xi \in (-\infty, -1/4) \cup (2, +\infty)$, corresponding asymptotic approximations can be characterized with an $N(\Lambda)$-solitons with diverse residual error order $\mathcal{O}(t^{-1+2\rho})$; The phase function $\theta(z)$ has four phase points and eight phase points on the jump contour in the space-time solitonic regions $\xi \in (0,2)$ and $\xi \in (-1/4, 0)$, respectively. The corresponding asymptotic approximations can be characterized with an $N(\Lambda)$-soliton and an interaction term between soliton solutions and the dispersion term with diverse residual error order $\mathcal{O}(t^{-3/4})$. Our results also confirm the soliton resolution conjecture and asymptotically stability of the N-soliton solutions for the mCH equation.