FUDAN UNIVERSITY

Applied Math Ph.D. Seminar

Lagrangian Hadamard integrator for wave equations: an asymptotic approach to highly-oscillatory wave fields

Speaker: Yuxiao Wei (Fudan University)
Time: 2023-11-09, 16:10 to 17:00
Location: Rm 1801, Guanghua East Tower
Advisor: Jin Cheng (Fudan University)

Abstract: In the numerical simulation of highly-oscillatory wave fields, direct numerical methods, such as finite-difference or finite-element methods, may suffer from dispersion or pollution errors so that such methods require an enormous computational grid to resolve these oscillations. Alternative methods, such as geometrical-optics based asymptotic methods, have been sought to resolve these highly-oscillatory wave phenomena, where how to solve the caustics phenomenon in geometric optics becomes a challenging problem. We propose a novel Hadamard integrator for the self-adjoint time-dependent wave equation in an inhomogeneous medium. We create a new asymptotic series based on the Gelfand-Shilov function, dubbed Hadamard's ansatz, to approximate the Green's function of the wave equation. Incorporating the leading term of Hadamard's ansatz into the Kirchhoff-Huygens representation, we develop an original Hadamard integrator for the Cauchy problem of the time-dependent wave equation and derive the corresponding Lagrangian formulation in geodesic polar coordinates. Equipped with low-rank representations, we apply the Hadamard integrator to efficiently solve time-dependent wave equations with highly oscillatory initial conditions. By judiciously choosing the medium-dependent time step, our new Hadamard integrator can propagate wave field beyond caustics implicitly and advance spatially overturning waves in time naturally.