SOC-MartNet: A Martingale Neural Network for the Hamilton-Jacobi-Bellman Equation without Explicit $\inf_u H$ in Stochastic Optimal Controls

Speaker: Shuixin Fang (Chinese Academy of Sciences)
Time: 2024-09-12, 16:10 to 17:00
Location: Rm 1801, Guanghua East Tower
Advisor: Tao Zhou (Chinese Academy of Sciences)

Abstract: In this talk, we introduce a martingale-based neural network, SOC-MartNet, for solving high-dimensional Hamilton-Jacobi-Bellman (HJB) equations where no explicit expression is needed for the infimum of the Hamiltonian, $\inf_u H(t, x, u, z, p)$, and stochastic optimal control problems (SOCPs) with controls on both drift and volatility. We reformulate the HJB equations for the value function by training two neural networks, one for the value function and one for the optimal control with the help of two stochastic processes - a Hamiltonian process and a cost process. The control and value networks are trained such that the associated Hamiltonian process is minimized to satisfy the minimum principle of a feedback SOCP, and the cost process becomes a martingale, thus, ensuring the value function network as the solution to the corresponding HJB equation. Moreover, to enforce the martingale property for the cost process, we employ an adversarial network and construct a loss function characterizing the projection property of the conditional expectation condition of the martingale. Numerical results show that the proposed SOC-MartNet is effective and efficient for solving HJB-type equations and SOCPs with a dimension up to 2000 in a small number of iteration steps (less than 2000) of training.