The condition for constructing a finite element from a superspline

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Abstract: In this work, we address a sufficient and necessary continuity condition for the construction of C^r conforming finite element spaces on general triangulations. It has been commonly conjectured that such spaces can be generated using the piecewise polynomials with degrees $\geq 2^d r + 1$ and an additional $C^{2^{d-s} r}$ smoothness on s-dimensional subsimplices. Under these conditions, three authors first provided a rigorous construction for any continuity in any dimension. We will prove that this condition is a tight condition for finite element construction. Specifically, we introduce the concept of extendability for the pre-element space, a generalization of (super)spline spaces and finite element spaces. We show that the superspline space is extendable if and only if such a condition holds, while the finite element space is always extendable under mild conditions. The theory is then established by combining both directions. This concept of extendability not only clarifies the essential connection between spline methods and finite element methods, but also provides valuable insights into the fundamental requirements for constructing conforming finite element spaces on general triangulations.