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Applied Math Ph.D. Seminar

Sharp Asymptotic Stability of Blasius Profile in the Steady Prandtl Equation

Speaker: Cheng Yuan (Fudan University)

Time: 2025-02-20, 16:10 to 17:00

Location: Rm 1801, Guanghua East Tower

Advisor: Zhen Lei (Fudan University)

Abstract: In this talk, I present an asymptotic stability result concerning the self-similar Blasius profiles $[\bar{u}, \bar{v}]$ of the stationary Prandtl boundary layer equation. Initially demonstrated by Serrin (1967, Proc. R. Soc. Lond), the profiles $[\bar{u}, \bar{v}]$ were shown to act as a self-similar attractor of solutions $[u, v]$ to the Prandtl equation through the use of von Mises transform and maximal principle techniques. Specifically, as $x \to \infty$, $\|u - \bar{u}\|_{L^\infty_y} \to 0$. Iyer (2020, ARMA) employed refined energy methods to derive an explicit convergence rate for initial data close to Blasius. Wang and Zhang (2023, Math. Ann.) utilized barrier function methods, removing smallness assumptions but imposing stronger asymptotic conditions on the initial data. It was suggested that the optimal convergence rate should be $\|u - \bar{u}\|_{L^\infty_y} \lesssim (x+1)^{-\frac{1}{2}}$, treating the stationary Prandtl equation as a 1-D parabolic equation in the entire space.

In our work, we establish that $\|u - \bar{u}\|_{L^\infty_y} \lesssim (x+1)^{-1}$. Our proof relies on discovering nearly conserved low-frequency quantities and inherent degenerate structures at the boundary, which enhance the convergence rate through iteration techniques. Notably, the convergence rate we have demonstrated is optimal. We can find special solutions of Prandtl's equation such that the convergence between the solutions and the Blasius profile is exact, represented as $(x+1)^{-1}$. This is a joint work with Prof. Hao Jia and Prof. Zhen Lei.